Assembly in Surgery

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1. Introduction

In [Y], I discussed glueing and splitting operations of geometric quadratic Poincaré complexes, and studied the $L^{-\infty}$-theory assembly map

$$A \ : \ H_*(X; \mathbb{L}^{-\infty}(p : E \to X)) \to L^{-\infty}(\pi_1 E)$$

for certain polyhedral stratified systems of fibrations $p : E \to X$, following the general description of assembly maps by Quinn [Q, §8]. This assembly map was constructed in two steps; first we used the gluing operation to construct a map

$$\alpha \ : \ H_*(X; \mathbb{L}^{-\infty}(p : E \to X)) \to L_*^{-\infty}(p)$$

from the homology to the controlled L-group, and then composed it with the forget-control map

$$F \ : \ L_*^{-\infty}(p : E \to X) \to L_*^{-\infty}(E \to \{*\}) = L_*^{-\infty}(\pi_1 E) \ .$$

The following was claimed in (3.9) of [Y].

Theorem. *If $p : E \to X$ is a polyhedral stratified system of fibrations on a finite polyhedron X, then the map α is an isomorphism.*

The map α was constructed in the following way: an element of $H_k(K; \mathbb{L}^{-\infty}(p : E \to X))$ can be thought of as a PL-triangulation V of the product $S^N \times \Delta^k$ of a shpere S^N (N large) and the k-somplex Δ^k together with

1. a simplicial map $\phi : V \to X$, and
2. a compatible family $\{\rho(\Delta) \mid \Delta \in V\}$, where $\rho(\Delta)$ is a quadratic Poincaré $(\dim \Delta + 2)$-ad on the pullback q of $\bar{p} : \mathbb{R}^l \times E \to E \to X$ via the map $\Delta \to V \to X$, and $\rho(\Delta)$ is 0 if Δ is a simplex in the boundary.

I claimed that these ads $\rho(\Delta)$'s can be glued together to give a geometric quadratic Poincaré complex on q:

Theorem (Glueing over a manifold) [Y, 2.10] *Let L be the barycentric subdivision of a PL-triangulation K of a compact n-dimensional manifold M possibly with a non-empty boundary ∂M and $p : E \to M$ be a map. And suppose each n-simplex $\Delta \in L$ is given an m-dimensional geometric quadratic Poincaré $(n+2)$-ad on $(p^{-1}(\Delta), p^{-1}(\partial_* \Delta))$ which are compatible on common faces. Then one can glue them together to get an m-dimensional geometric quadratic Poincaré pair on $(E, p^{-1}(\partial M))$.*

If this is possible, then its functorial image on $\bar{p}$ gives a geometric quadratic complex on $\bar{p}$. By the 'barycentric subdivision argument' [Y, p.589], this assembled complex is equivalent to arbitrarily small complex and defines an element of $L_*^{-\infty}(p)$.

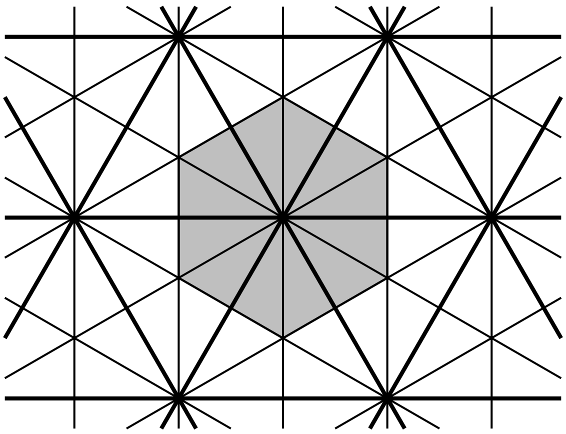
Unfortunately the argument given in [Y] is insufficient to prove this. The aim of this short note is to describe how to remedy this.

2. Glueing over a manifold

In [Y], I described the glueing operation of two quadratic Poincaré pairs along a common codimension 0 subcomplex of the boundaries. If there is an order of the n-simplices $\Delta_1, \ldots, \Delta_r$ of L so that $(\Delta_1 \cup \ldots \cup \Delta_i) \cap \Delta_{i+1}$ is the union of $(n-1)$-simplices for each i, then we can successively glue the pieces in this linear order. But this seems very difficult to achieve. The strategy used in [Y] is the following:

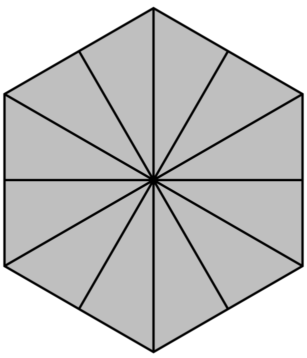
> *For each vertex v of K, consider its star $S(v)$ in L, i.e. the dual cone of v. Two such dual cones are either disjoint or meet along codimension 1 cells. The glueing problem over $S(v)$ can be solved by looking at the link $L(v)$ of v in L. Note that $L(v)$ is an $(n-1)$-dimensional sphere or disk and the triangulation is the first barycentric subdivision of another. Thus we can keep on reducing the dimension until the link becomes a circle or an arc, and in this case there is an obvious order of 2-simplices and glueing can be done.*

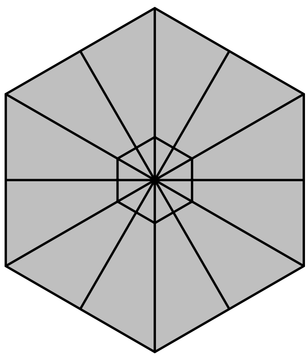
The fact is that the induction fails, since any two n-simplices of $S(v)$ have the vertex v in common and are never disjoint.



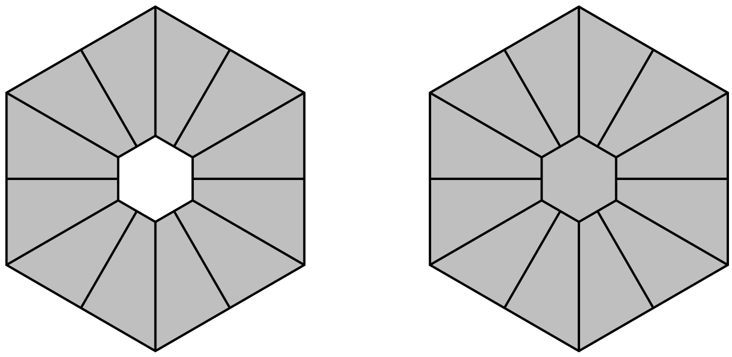
There are two possible remedies for this. The first one is to use a different definition for the homology groups. This was actually done in [R].

Here I propose another remedy. Let us look at the dual cone at the vertex v. Let c denote the quadratic Poincaré complex lying over v. Split each of the pieces of the dual cone so that the pieces near v are of the form $c \otimes$ (a small simplex):





Here we do not need stabilization to split. We would like to glue the pieces away from v first, and then fill in the hole with a piece of the form $c \otimes$ (a small copy of the dual cone):



To carry out the induction steps, we need to deal with holes of more complicated forms, and I have not worked out the details yet.

Remarks. (1) The control map should be a polyhedral stratified system of fibrations.
(2) The picture above may be misleading. The 'hole' itself lies over the vertex v, because $c \otimes$ (a small copy of the dual cone) can only live over v.
(3) Splitting needs a similar treatment.

References

[Q] F. Quinn, Ends of Maps II, *Invent. math.* **68**, 353–424 (1982).

[R] A. Ranicki, *Algebraic L-theory and Topological Manifolds*, Cambridge Tracts in Mathematics **102**, Cambridge Univ. Press (1992).

[Y] M. Yamasaki, L-groups of crystallographic groups, *Invent. math.* **88**, 571–602 (1987).