HYPERBOLIC KNOTS AND 4-DIMENSIONAL SURGERY

MASAYUKI YAMASAKI

1. Introduction

In [5], Hegenbarth and Repovš used controlled surgery exact sequence of [6] to show that the surgery obstruction theory works for certain 4-manifolds without assuming that the fundamental groups are good. Among their examples are 4-manifolds whose fundamental groups are knot groups. Let K be a knot in S^3, and let $E(K)$ dcenote its exterior. Let $M(K)$ denote the 4-manifold $\partial(E(K) \times D^2)$. Hegenbarth and Repovš showed that the surgery obstruction theory works in the topological category when K is a torus knot. The aim of this short note is to show that their strategy also works when K is a hyperbolic knot:

Theorem 1. *The TOP-surgery sequence*

$$\mathcal{S}(M(K)) \longrightarrow [M(K), G/TOP] \longrightarrow L_4(\pi_1(M(K)))$$

is exact when K is a hyperbolic knot.

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2. Proof of Theorem 1

Let K be a hyperbolic knot in S^3. Consider the Epstein-Penner canonical decompsition of $S^3 - K$ into ideal polyhedra [3]. It induces a decomposition of $E(X)$ into truncated polyhedra. The cut-locus B with respect to the cusp is the dual of these decompositions, and is the spine for $E(K)$. B is also a spine of $E(X) \times D^2$, and the restriction π of the collapsing map $E(X) \times D^2 \to B$ to $M(K)$ is UV^1, since each point inverse is the union of finitely many copies of 2-discs whose boundaries are identified.

The rest of the proof is exactly the same as the one given in [5]. We give the outline for the convenience of the reader. Since the map $\pi : M(K) \to B$ is UV^1, there is a commutative diagram

$$\begin{array}{ccccc} \mathcal{S}_{\epsilon,\delta}(M(K)) & \longrightarrow & [M(K), G/TOP] & \longrightarrow & H_4(B; \mathbb{L}) \\ \downarrow & & \| & & \downarrow A \\ \mathcal{S}(M(K)) & \longrightarrow & [M(K), G/TOP] & \longrightarrow & L_4(\pi_1(B)) \end{array}$$

for sufficiently small $\epsilon \gg \delta > 0$. The first row is the controlled surgery sequence with trivial local fundamental groups and is exact [6]. The second row is the ordinary surgery sequence we are interested in.

The assembly map A for B is an isomorphism. This can be observed in the following way. Recall that B is a homology circle; let $\phi : B \to S^1$ be a homology equivalence. This map induces a commutative diagram whose top arrow is an isomorphism.

$$\begin{array}{ccc} H_4(B;\mathbb{L}) & \xrightarrow[\cong]{\phi_*} & H_4(S^1;\mathbb{L}) \\ \downarrow A & & \downarrow A \\ L_4(\pi_1(B)) & \xrightarrow{\phi_*} & L_4(\pi_1(S^1)) \end{array}$$

Arvinda, Farrell, and Roushon showed that the bottom row is also an isomorphism [1], and the assembly map A for S^1 has been known to be an isomorphism [2] [4]. Therefore, the assembly map $A : H_4(B;\mathbb{L}) \to L_4(\pi_1(B))$ for B is also an isomorphism.

Now a simple diagram chase shows that the ordinary surgery sequence is also exact. This completes the proof of Theorem 1.

References

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Department of Applied Science, Okayama University of Science, Okayama, Okayama 700-0005, Japan