Complex Hyperbolic Geometry and Related Topics
–Meeting in honor of Shigeyasu Kamiya's Retirement

Abstracts

On the limit set of complex Kleinian groups
José Seade

It is well-known that the action of a discrete subgroup Γ of $PSL(2,\mathbb{C})$ on the complex projective line $\mathbb{P}_{\mathbb{C}}{}^1$ splits this space into two invariant sets with remarkable properties, $\mathbb{P}_{\mathbb{C}}{}^1 = \Lambda(\Gamma) \cup \Omega(\Gamma)$. The first of these is the limit set $\Lambda(\Gamma)$, which is a minimal set for the action. It is there where the dynamics concentrates. This set either consists of at most two points, and the group is said to be elementary, or else it has infinite cardinality. For non-elementary groups one has that $\Lambda(\Gamma)$ is a perfect set and every orbit is dense. The second of these sets , which can be empty, is the largest set where the action is properly discontinuous. The quotient space $\Gamma/\Omega(\Gamma)$ is a Riemann surface equipped with the structure of a projective orbifold.

In this talk we shall discuss how these concepts and results extend to complex dimension two, *i.e.*, to discrete subgroups $PSL(2,\mathbb{C})$ acting on $\mathbb{P}_{\mathbb{C}}{}^2$.

A characterization of complex hyperbolic Kleinian groups with trace fields contained in $\mathbb{R}$
Joonhyung Kim

Let $\Gamma < \mathbf{SU}(2,1)$ be a non-elementary complex hyperbolic Kleinian group. The *trace field* of Γ is the field generated by the traces of all the elements of Γ over the base field $\mathbb{Q}$. In 1988, B. Maskit characterized non-elementary hyperbolic Kleinian groups of $\mathbf{SL}(2,\mathbb{C})$ whose trace fields are contained in $\mathbb{R}$. After that, this theorem has been generalized in $SU(n,1)$ and $Sp(n,1)$ cases. In 2012, X. Fu, L. Li and X. Wang generalized this theorem in $SU(2,1)$ case and I generalized further in $SU(3,1)$ case (joint work with Sungwoon Kim) and $Sp(2,1)$ case.

In this talk, I will explain previous results and present current results on most generel case, that is $SU(n,1)$ case for all $n \geq 2$. This is a joint work with Sungwoon Kim.

Non-Kähler complex geometric structures on homogeneous spaces
Keizo Hasegawa

In this survey talk we discuss non-Kähler complex geometric structures, such as pseudo-Kähler structures, locally comformally Kähler structures, HKT-structures, generalized Calabi-Yau structures on compact homogeneous and locally homogeneous manifolds. In the case of complex dimension two, we have fairly complete classifications of all those with such structures. The cohomological study is of central importance in studying them, in some way exending Kähler cases.

Parabolic isometries of hyperbolic spaces and discreteness
John Parker

In 1963 Shimizu gave a famous lemma about the discreteness of a subgroup of $PSL(2, \mathbb{R})$ containing a translation. This lemma has been generalised to isometries of higher dimensional real hyperbolic spaces by Leutbecher, Wielenberg and Waterman. In 1983 Kamiya gave a version of Shimizu's lemma for isometry groups of real and quaternionic hyperbolic spaces containing a vertical translation. Subsequently, several authors, including Kamiya, have given generalisations to isometry groups containing more general parabolic maps. I will survey these developments. I will then discuss recent joint work with Cao where we give a version of Shimizu's lemma for isometry groups of complex and quaternionic hyperbolic space in all dimensions containing a general parabolic map.

Parabolic quasiconformal conjugacy classes in the Heisenberg group
Youngju Kim

In this talk, we study noncompact 3-dimensional manifolds obtained by quotienting the Heisenberg group by cyclic groups of parabolic automorphisms. In particular, we consider the quasiconformal equivalence classes of such manifolds. This is related to the quasiconformal conjugacy classes of parabolic isometries acting on the complex hyperbolic plane. We here prove that the quotient of the Heisenberg group by the cyclic group generated by a screw parabolic automorphism is quasiconformally distinct from the one generated by a Heisenberg translation.

Polynomial automorphisms of $\mathbf{C}^n$ preserving the Markoff-Hurwitz polynomial
Hengnan Hu, Ser Peow Tan, and Ying Zhang

We will talk about the action of the group of polynomial automorphisms of $\mathbf{C}^n$ ($n \geq 3$) which preserve the Markoff-Hurwitz polynomial

$$H(\mathbf{x}) := x_1^2 + \cdots + x_n^2 - x_1 x_2 \dots x_n.$$

We will discuss the determination of the group and its action on $\mathbf{C}^n$; the description of a non-empty open subset of $\mathbf{C}^n$ on which the group acts properly discontinuously (domain of discontinuity); and identities for the orbit of points in the domain of discontinuity. This is joint work with Hengnan Hu and Ying Zhang.

Arithmetic aspects of growth rates for hyperbolic Coxeter groups
Yohei Komori

Growth functions of Coxeter groups are computable by means of Steinberg formula and many people studied growth rates of them numerically and theoretically. In this talk I will review arithmetic properties of growth rates for 2 and 3 dimensional hyperbolic Coxeter groups mainly, and report new results related to 2-Salem numbers.

Complex hyperbolic triangle groups of type (n, n, ∞)
Shigeyasu Kamiya

A complex hyperbolic triangle group is a group generated by three complex involutions fixing complex geodesics in complex hyperbolic 2-space $\mathrm{H}^2_{\mathbf{C}}$. In this talk, we discuss complex hyperbolic triangle groups of type (n, n, ∞). We make a list of non-discrete groups of type $(n, n, \infty; k)$. Especially, we show that if $n \geq 22$, then complex hyperbolic triangle groups of type $(n, n, \infty; k)$ are not discrete.

An additively closed subset of the Smith set in the real representation ring
Masaharu Morimoto

Let G be a finite group. Real G-modules V and W (of finite dimension) are called *Smith equivalent*, and written $V \sim_{Sm} W$, if there exists a homotopy sphere Σ with smooth G-action such that $\Sigma^G = \{a, b\}$, $T_a(\Sigma) \cong V$, and $T_b(\Sigma) \cong W$. This relation $\sim_{Sm}$ is an equivalence relation in the family of all real G-modules with exactly one G-fixed point. We call the set $\mathrm{Sm}(G)$ consisting of all elements $x = [V] - [W]$ in $\mathrm{RO}(G)$ such that $V \sim_{Sm} W$ the *Smith set* of G. It follows from works by Bredon and Cappell-Shaneson that $\mathrm{Sm}(G)$ is not an additively closed subset of $\mathrm{RO}(G)$ if G has a normal subgroup N such that the order of the quotient G/N is 8. Let $\mathrm{Sm}(G)_{\mathcal{P}}$ denote the set consisting of all $x = [V] - [W] \in \mathrm{Sm}(G)$ such that $\mathrm{res}^G_P V \cong \mathrm{res}^G_P W$ for all Sylow subgroups P. We call $\mathrm{Sm}(G)_{\mathcal{P}}$ the *primary Smith set* of G. It follows from works by Bredon, Petrie, and Cappell-Shaneson that the set $\mathrm{Sm}(G) \smallsetminus \mathrm{Sm}(G)_{\mathcal{P}}$ is a finite set (possibly, the empty set). It has been observed that for various groups G, $\mathrm{Sm}(G)_{\mathcal{P}}$ are subgroups of $\mathrm{RO}(G)$. But we recently found that there are finite groups G such that $\mathrm{Sm}(G)_{\mathcal{P}}$ are not additively closed in $\mathrm{RO}(G)$. There arise a problem to find an additively closed subset of $\mathrm{RO}(G)$ included in $\mathrm{Sm}(G)_{\mathcal{P}}$ and occupying a large portion of $\mathrm{Sm}(G)_{\mathcal{P}}$. We would like to discuss this problem and gives an answer to it by means of one-fixed point actions on spheres. Using the answer, we determine $\mathrm{Sm}(G)_{\mathcal{P}}$ for various finite Oliver groups G in terms of algebra.

Deformations of hyperbolic structures, nontrivial 4-cobordisms and homomorphisms of their fundamental groups

Boris N. Apanasov

We discuss how the global geometry and topology of a non-trivial compact 4-dimensional cobordism M (cf. [1]) whose interior has a complete hyperbolic structure depend on properties of the variety of discrete representations of the fundamental group of its boundary ∂M -cf. [2, 3] and on different ergodic actions of a uniform hyperbolic lattice.

In addition to the standard conformal ergodic action of a uniform hyperbolic lattice on the round sphere S^{n-1} and its quasiconformal deformations in S^n, we present several constructions of unusual actions of such lattices on everywhere wild spheres (boundaries of quasisymmetric embeddings of the closed n-ball into S^n), on non-trivial $(n-1)$-knots in S^{n+1}, as well as actions defining non-trivial compact cobordisms with complete hyperbolic structures in its interiors. Such unusual actions correspond to discrete representations of a given hyperbolic lattice from "non-standard" components of its varieties of representations (faithful or with large kernels of defining homomorphisms).

We construct such non-trivial hyperbolic 4-cobordisms M whose boundary components have a great geometric symmetry. These 3-dimensional boundary manifolds are covered by the discontinuity set $\Omega(G) \subset S^3$ with two connected components Ω_1 and Ω_2, where the action Γ of the fundamental group $\pi_1(\partial M)$ is symmetric and has contractible fundamental polyhedra of the same combinatorial (3-hyperbolic) type (cf. [4]). Nevertheless we show that a geometric symmetry of boundary components of our hyperbolic 4-cobordism $M(G)$) are not enough to ensure that the group $G = \pi_1(M)$ is quasi-Fuchsian, and in fact our 4-cobordism M is non-trivial, see [5]. This is related to non-connectedness of the variety of discrete representation of the uniform hyperbolic lattice Γ and infinite kernels of the constructed homomorphisms $\Gamma \to G$ (similar to homomorphisms in Thurston's non-rigidity of deformations of hyperbolic manifolds). This gives also a new view on Andreev's hyperbolic polyhedron theorem.

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[2] Boris Apanasov, *Nonstandard uniformized conformal structures on hyperbolic manifolds.* - Inventiones Mathematicae, **105:1** (1991), 137-152.

[3] Boris Apanasov, *Conformal geometry of discrete groups and manifolds.* - De Gruyter Expositions in Math. **32**, W. de Gruyter, Berlin - New York, 2000, XIV + 523 pp.

[4] Boris Apanasov, *Quasisymmetric embeddings of a closed ball inextensible in neighborhoods of any boundary points*, Ann. Acad. Sci. Fenn., Ser. AI **14:2** (1989), 243-255.

[5] Boris Apanasov, *Group homomorphisms with infinite kernel and hyperbolic 4-cobordisms.* - Computational and Geometric Topology, Universitá di Modena e Reggio Emilia di Pubblicazione, Modena, 2014, 11-24.