ON THE JOHNSON FILTRATION OF THE AUTOMORPHISM GROUP OF A FREE GROUP

TAKAO SATOH

For a free group F_n with basis $x_1, \ldots, x_n$, set $H := F_n^{\mathrm{ab}}$ the abelianization of F_n. The kernel of the natural homomorphism $\rho : \mathrm{Aut}\, F_n \to \mathrm{Aut}\, H$ induced from the abelianization of $F_n \to H$ is called the IA-automorphism group of F_n, and denoted by IA_n. The IA-automorphism group IA_n reflects much richness and complexity of the structure of $\mathrm{Aut}\, F_n$. In particular, it plays an important role on the study of (co)homology of $\mathrm{Aut}\, F_n$.

For the lower central series $\Gamma_n(k)$ of F_n, the action of $\mathrm{Aut}\, F_n$ on the nilpotent quotient $F_n/\Gamma_n(k)$ induces a natural homomorphism $\mathrm{Aut}\, F_n \to \mathrm{Aut}\,(F_n/\Gamma_n(k+1))$. Then its kernel $\mathcal{A}_n(k)$ defines a descending central filtration $\mathrm{IA}_n = \mathcal{A}_n(1) \supset \mathcal{A}_n(2) \supset \cdots$. This filtration is called the Johnson filtration of $\mathrm{Aut}\, F_n$. Its graded quotients $\mathrm{gr}^k(\mathcal{A}_n) := \mathcal{A}_n(k)/\mathcal{A}_n(k+1)$ are considered as one by one approximations of IA_n, and they have much important information of IA_n.

In order to study the $\mathrm{GL}(n, \mathbf{Z})$-module structure of $\mathrm{gr}^k(\mathcal{A}_n)$, the Johnson homomorphisms

$$\tau_k : \mathrm{gr}^k(\mathcal{A}_n) \hookrightarrow \mathrm{Hom}_{\mathbf{Z}}(H, \mathcal{L}_n(k+1))$$

of $\mathrm{Aut}\, F_n$ are defined. The purpose of our research is to clarify the structure of the image of τ_k. In general, however, it is quite difficult problem to determine even the rank of the image of τ_k.

Now, we consider the lower central series $\mathcal{A}'_n(1), \mathcal{A}'_n(2), \ldots$ of IA_n. Since the Johnson filtration is central, $\mathcal{A}'_n(k) \subset \mathcal{A}_n(k)$ for each $k \geq 1$. It is conjectured that $\mathcal{A}'_n(k) = \mathcal{A}_n(k)$ for each $k \geq 1$ by Andreadakis who showed $\mathcal{A}'_2(k) = \mathcal{A}_2(k)$ and $\mathcal{A}'_3(3) = \mathcal{A}_3(3)$. Set $\mathrm{gr}^k(\mathcal{A}'_n) := \mathcal{A}'_n(k)/\mathcal{A}'_n(k+1)$. Then we can define a $\mathrm{GL}(n, \mathbf{Z})$-equivariant homomorphism

$$\tau'_k : \mathrm{gr}^k(\mathcal{A}'_n) \to H^* \otimes_{\mathbf{Z}} \mathcal{L}_n(k+1)$$

by the same way as τ_k. In this talk, we determine the cokernel of τ'_k for any $k \geq 2$ and $n \geq k+2$, and give an upper bound on the cokernel of τ_k.

Department of Mathematics, Kyoto University, Kitashirakawaoiwake-cho, Sakyo-ku, Kyoto city, 606-8502, Japan.
E-mail address: takao@math.kyoto-u.ac.jp