SMITH SET FOR A FINITE PERFECT GROUP

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Abstract. Let $G$ be a finite group. Two $G$-modules $U$ and $V$ are called Smith equivalent if there is a smooth action on a sphere with just two fixed points $x$ and $y$ such that $U$ (resp. $V$) is equivalent to the tangential $G$-module over $x$ (resp. $y$). A Smith set $Sm(G)$ is a subset of the real representation ring $RO(G)$ consisting of all $U - V$ such that $U$ and $V$ are Smith equivalent $G$-modules.

Now we let $G$ be a finite perfect group. It is completely known a perfect group $G$ so that $Sm(G)$ is not trivial. Let $P(G)$ be the set of subgroups of $G$ of prime power order and $P_{\text{odd}}(G)$ the set of subgroups of odd prime power order. Further let $RO(G)_P^{(G)}$ be the subgroup of $RO(G)$ consisting of $U - V$ with $\dim(U) = \dim(V)$. For a set $\mathcal{F}$ of subgroups of $G$, we denote by $RO(G)_\mathcal{F}^{(G)}$ the subgroup of $RO(G)_P^{(G)}$ consisting of $U - V$ such that $U$ and $V$ are isomorphic as a $P$-module for any $P \in \mathcal{F}$. Then $RO(G)_{P(G)}^{(G)} \subset Sm(G) \subset RO(G)_{P_{\text{odd}}(G)}^{(G)}$. Further if a finite perfect group $G$ has no element of order 8 then $Sm(G) = RO(G)_{P(G)}^{(G)}$. In this talk we treat finite perfect groups $G$ of small order and discuss whether $Sm(G) = RO(G)_{P(G)}^{(G)}$.

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