

# EQUIVARIANT SURGERY UNDER THE WEAK GAP CONDITION

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**Abstract.** In this article we explain the equivariant surgery obstruction group with middle dimensional singular set and hypercomputability of this group.

## 1. INTRODUCTION

In this article we describe an equivariant surgery theory under the ‘weak gap condition’ obtained in joint research with Anthony Bak.

Throughout this paper, let  $G$  be a finite group and let  $\mathcal{S}(G)$  denote the set of all subgroups of  $G$ . Let  $X$  be a compact smooth  $G$ -manifold  $X$  of dimension  $n$ . The *singular set*  $X_{\text{sing}}$  of  $X$  is the subset  $\bigcup_{g \in G \setminus \{e\}} X^g$  of  $X$  and the free part  $X_{\text{free}}$  of  $X$  is the complement of  $X_{\text{sing}}$  in  $X$ . Let  $\widehat{\Pi}(G, X)$  denote the set of all connected components of the fixed point manifolds  $X^H$ , where  $H$  runs over the set  $\mathcal{S}(G)$ . A precise definition of  $\widehat{\Pi}(G, X)$  will be given in the next section. The underlying manifold of an element  $t$  in  $\widehat{\Pi}(G, X)$  is denoted by  $X_t$ . The map  $\rho_X : \widehat{\Pi}(G, X) \rightarrow \mathcal{S}(G)$  is defined by

$$\rho_X(t) = \bigcap_{x \in X_t} G_x$$

where  $G_x$  is the isotropy subgroup at  $x$  in the  $G$ -manifold  $X$ . Clearly  $\widehat{\Pi}(G, X)$  inherits a  $G$ -action from  $X$ . For an integer  $i$ , let  $\widehat{\Pi}(G, X, i)$  denote the subset of  $\widehat{\Pi}(G, X)$  consisting of all  $t$  such that  $\dim X_t = i$ .

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If  $X$  is a connected, oriented smooth  $G$ -manifold then we have the *orientation homomorphism*  $w_X : G \rightarrow \{\pm 1\}$  associated with  $X$ , namely  $w_X(g) = 1$  if and only if  $g : X \rightarrow X$  is orientation preserving. The group ring  $\mathbb{Z}[G]$  has the (anti-)involution – defined by

$$\left( \sum_{g \in G} a_g g \right)^{-} = \sum_{g \in G} w_X(g) a_g g^{-1},$$

where  $a_g$  are integers.

We say that  $X$  satisfies the *weak gap condition* (resp. *gap condition*, *strong gap condition*) for  $\{e\}$  if  $2 \dim X^g \leq n$  (resp.  $2 \dim X^g + 1 \leq n$ ,  $2 \dim X^g + 2 \leq n$ ) for any  $g \in G$  with  $g \neq e$ . Equivariant surgery theories under the strong gap condition and the gap condition are discussed in, for example, [18] and [9, 10], respectively. In [9, 10], the set

$$Q_X = \{g \in G \mid g^2 = e, g \neq e, \dim X^g = [(\dim X - 1)/2]\},$$

where  $[r]$  denotes the greatest integer not exceeding  $r$ , has the role of generating a quadratic form parameter: namely

$$\Lambda(Q_X) = \{x - (-1)^k \bar{x} \mid x \in \mathbb{Z}[G]\} + \mathbb{Z}[Q_X]$$

is a form parameter in the sense of [1]. It was found in [9] that the surgery obstruction group under the gap condition  $\{e\}$  depends on  $\Lambda(Q_X)$ .

To discuss the essential part of our equivariant surgery theory under the weak gap condition for  $\{e\}$ , let us assume  $\dim X = n = 2k$  (even)  $\geq 6$ . Under this assumption, we set  $\Theta_X = \widehat{\Pi}(G, X, k)$ . Then  $\Theta_X$  is the disjoint union of  $\Theta_{X,+} = \Theta_+(G, X)$  and  $\Theta_{X,-} = \Theta_-(G, X)$  such that

$$\Theta_{X,+} = \{t \in \Theta_X \mid X_t \text{ is orientable}\},$$

$$\Theta_{X,-} = \{t \in \Theta_X \mid X_t \text{ is nonorientable}\}.$$

If  $X$  is a compact smooth  $G$ -manifold, we denote by  $\widetilde{\Theta}_{X,+} = \widetilde{\Theta}_+(G, X)$  the set of all generators in  $H_k(X_t, \partial X_t; \mathbb{Z}) \cong \mathbb{Z}$ , where  $t$  runs over  $\Theta_{X,+}$ . If  $\omega$  is a generator in  $H_k(X_t, \partial X_t; \mathbb{Z})$  then so is  $-\omega$ . Thus we have a bijection from  $\widetilde{\Theta}_{X,+}$  to  $\Theta_{X,+} \times \{\pm 1\}$ . In addition,  $\widetilde{\Theta}_{X,+}$  has a canonical  $\{\pm 1\}$ -action. Furthermore, we can give a  $G$ -action to  $\widetilde{\Theta}_{X,+}$  so that the projection map  $\pi_X : \widetilde{\Theta}_{X,+} \rightarrow \Theta_{X,+}$  is  $G$ -equivariant. Set

$$S_X = \{g \in G \mid g^2 = e, g \neq e, \dim X^g = k\}.$$

Note that this set is also determined by  $\rho_X : \Theta_X \rightarrow \mathcal{S}(G)$ . We assign the datum

$$\mathcal{D}_X = (G, (-1)^k, w_X, \pi_X : \tilde{\Theta}_{X,+} \rightarrow \Theta_X, \rho_X : \Theta_X \rightarrow \mathcal{S}(G), Q_X)$$

to a compact, connected, oriented, smooth  $G$ -manifold of dimension  $n = 2k \geq 6$  satisfying the weak gap condition for  $\{e\}$ . Note that if  $X$  satisfies the strong gap condition or the gap condition for  $\{e\}$  then  $\mathcal{D}_X$  is essentially the datum

$$(G, (-1)^k, w_X) \text{ or } (G, (-1)^k, w_X, Q_X), \text{ respectively.}$$

By [4], the datum  $\mathcal{D}_X$  provides a Witt group  $\nabla W(\mathcal{D}_X)_{\text{proj}}$  and this group is the surgery obstruction group under the weak gap condition for  $\{e\}$ . In the case where  $X$  fulfills the strong gap condition or the gap condition for  $\{e\}$  then the group  $\nabla W(\mathcal{D}_X)_{\text{proj}}$  coincides with a Wall group  $L_n^h(\mathbb{Z}[G], w)_{\text{proj}}$  or a Bak group  $W_n(\mathbb{Z}[G], \Lambda(Q_X), w)_{\text{proj}}$ , respectively. The group  $\nabla W(\mathcal{D}_X)_{\text{proj}}$  consists of equivalence classes of tuples  $(M, B, q, \alpha)$  such that  $M$  is a finitely generated projective  $\mathbb{Z}[G]$ -module,  $B$  is a nonsingular  $(-1)^k$ -Hermitian form  $M \times M \rightarrow \mathbb{Z}[G]$ ,  $q$  is a generalized quadratic form  $M \rightarrow \mathbb{Z}[G]/(\Lambda(Q_X) + \mathbb{Z}[S_X])$ , and  $\alpha$  is a pair consisting of a  $G \times \{\pm 1\}$ -map  $\tilde{\alpha}_+ : \tilde{\Theta}_{X,+} \rightarrow M$  and  $G$ -map  $\alpha : \Theta_X \rightarrow M/2M$  making the diagram

$$\begin{array}{ccc} \tilde{\Theta}_{X,+} & \xrightarrow{\tilde{\alpha}_+} & M \\ \pi_X \downarrow & & \downarrow \\ \Theta_X & \xrightarrow{\alpha} & M/2M \end{array}$$

commutative. We will give the precise definition of  $\nabla W(\mathcal{D}_X)_{\text{proj}}$  in Section 3.

A  $G$ -framed map  $\mathbf{f} = (f, b)$  of degree one is a pair consisting of a degree one  $G$ -map  $f : (X, \partial X) \rightarrow (Y, \partial Y)$ , where  $X$  and  $Y$  are compact, connected, oriented, smooth  $G$ -manifolds and a  $G$ -vector bundle isomorphism  $b : T(X) \oplus f^*\eta \rightarrow f^*\xi$ , where  $T(X)$  is the tangent bundle of  $X$  and  $\xi, \eta$  are real  $G$ -vector bundles over  $Y$ .

**Theorem 1.1** ([4]). *Let  $\mathbf{f} = (f, b)$  be a  $G$ -framed map of degree one as above. Suppose the following conditions are satisfied.*

- (1)  $\dim X = \dim Y = n = 2k \geq 6$  is even.
- (2)  $X$  satisfies the weak gap condition for  $\{e\}$ .
- (3)  $\dim(X_t \cap X_{t'}) \leq k - 2$  for all  $t \in \hat{\Pi}(G, X, k)$  and  $t' \in \hat{\Pi}(G, X, k - 1)$ .
- (4)  $Y$  is simply connected.

(5)  $\partial f : \partial X \rightarrow \partial Y$  is  $\mathbb{Z}$ -homology equivalence.

(6)  $f^P : X^P \rightarrow Y^P$  is a  $\mathbb{Z}_p$ -homology equivalence for any prime  $p$  and any  $P \in \mathcal{S}(G)$  of  $p$ -power order  $\neq 1$ .

Then an element  $\sigma(\mathbf{f})$  in  $\nabla W(\mathcal{D}_X)_{\text{proj}}$  is assigned to  $\mathbf{f} = (f, b)$  so that  $\sigma(\mathbf{f}) = 0$  if and only if  $\mathbf{f}$  is  $G$ -framed cobordant, by  $G$ -surgeries on  $\mathbf{f}$  relative to the singular set and the boundary of  $X$ , to a  $G$ -framed map  $\mathbf{f}' = (f', b')$  such that the ambient map  $f' : X' \rightarrow Y$  is a homotopy equivalence.

Let  $X$  be a smooth  $G$ -manifold as in the theorem above. For  $H \in \mathcal{S}(G)$ , we obtain the datum

$$(1.1) \quad \mathcal{D}_H = (H, (-1)^k, w_H, \pi_H : \tilde{\Theta}_H \rightarrow \Theta_H, \rho_H : \Theta_H \rightarrow \mathcal{S}(H), Q_H)$$

by setting  $\mathcal{D}_H = \mathcal{D}_{\text{res}_H^G X}$ ,  $w_H = w_{\text{res}_H^G X}$ ,  $\tilde{\Theta}_{H,+} = \tilde{\Theta}_{\text{res}_H^G X,+}$ ,  $\Theta_H = \Theta_{\text{res}_H^G X}$ ,  $\pi_H = \pi_{\text{res}_H^G X}$ ,  $\rho_H = \rho_{\text{res}_H^G X}$ , and  $Q_H = Q_{\text{res}_H^G X}$ . By definition, we have

$$\begin{aligned} \Theta_H &= \{t \in \Theta_G \mid \rho_G(t) \cap H \neq \{e\}\}, \\ \tilde{\Theta}_{H,+} &= \{\omega \in \tilde{\Theta}_{G,+} \mid \rho_G \circ \pi_G(\omega) \neq \{e\}\}, \\ Q_H &= Q_X \cap H. \end{aligned}$$

Set  $S_H = S_{\text{res}_H^G X}$ . Then we have

$$S_H = S_X \cap H.$$

**Lemma 1.2.** *In the above setting and notation, if*

(C1)  $\rho_G(t)$  has prime order for each  $t \in \Theta_G$

then the equality

$$\Theta_{H \cap K} = \Theta_H \cap \Theta_K$$

holds for all  $H, K \in \mathcal{S}(G)$ .

The next theorem is proved by using results in [19], [6], [1], [2].

**Theorem 1.3.** *Let  $X$  be as in Theorem 1.1 and  $\mathcal{F}$  a subset of  $\mathcal{S}(G)$  closed with respect to conjugation and intersections. Suppose  $\mathcal{F}$  contains all maximal cyclic subgroups of  $G$ . Further suppose  $X$  satisfies the following.*

(C1)  $\rho_G(t)$  has prime order for each  $t \in \Theta_G$ .

$$(C2) \quad \Theta_G \times \Theta_G = \bigcup_{H \in \mathcal{F}} \Theta_H \times \Theta_H.$$

Then  $\nabla W(\mathcal{D}_G)_{\text{proj}}$  is  $\mathcal{F}$ -hypercomputable; in particular,

$$\text{Ind} : \lim_{\rightarrow \tilde{\mathcal{F}}} \nabla W(\mathcal{D}_-)_{\text{proj}} \rightarrow \nabla W(\mathcal{D}_G)_{\text{proj}} \quad \text{and} \quad \text{Res} : \nabla W(\mathcal{D}_G)_{\text{proj}} \rightarrow \lim_{\leftarrow \tilde{\mathcal{F}}} \nabla W(\mathcal{D}_-)_{\text{proj}}$$

are isomorphisms, where

$$\tilde{\mathcal{F}} = \{K \in \mathcal{S}(G) \mid \exists H \in \mathcal{F} : H \trianglelefteq K, K/H \text{ has prime power order}\}.$$

The reader can obtain basic knowledge of the Burnside ring from [6], [5], [12]. A finite group  $G$  is called an *Oliver group* if  $G$  admits a smooth  $G$ -action on a disk without  $G$ -fixed points, cf. [16, 15], [8].

**Theorem 1.4.** *Let  $G$  be an Oliver group and let  $X$  be as in the above theorem. Let  $D$  be an acyclic finite  $G$ -CW complex such that*

$$(C3) \quad \text{the Euler characteristics } \chi(D^K) \text{ are equal to 1 for all subgroups } K \text{ of the group } \langle \rho_G(t), \rho_G(t') \rangle, \text{ where } t, t' \text{ range over } \Theta_G.$$

Then the vanishing property

$$([G/G] - [D])^{2m+2} \nabla W(\mathcal{D}_G)_{\text{proj}} = 0$$

holds for the integer  $m$  defined by  $|G| = 2^m m'$  with odd  $m'$ , and where  $[G/G]$  and  $[D]$  are the elements in the Burnside ring determined respectively by the finite  $G$ -CW complexes  $G/G$  and  $D$ .

## 2. CONNECTED COMPONENTS OF FIXED POINT SETS

Let  $X$  be a finite  $G$ -CW complex or a compact smooth  $G$ -manifold. According to [17], we define the  $G$ -poset  $\Pi(G, X)$  associated with  $X$  by

$$\Pi(G, X) = \coprod_{H \in \mathcal{S}(G)} \pi_0(X^H),$$

where  $\pi_0(X^H)$  is the set of all connected components of the  $H$ -fixed point set  $X^H$  of  $X$ . The map  $\rho : \Pi(G, X) \rightarrow \mathcal{S}(G)$  is defined so that for  $t \in \Pi(G, H)$ ,  $\rho(t) = H$  holds if and only if  $t \in \pi_0(X^H)$ . The underlying space of  $t \in \Pi(G, X)$  is denoted by  $X_t$ . The set  $\Pi(G, X)$  inherits a  $G$ -action from  $X$ , namely for  $g \in G$  and  $t \in \Pi(G, H)$ ,

$gt$  is the element having the property  $\rho(gt) = g\rho(t)g^{-1}$  and  $X_{gt} = gX_t$ . The set  $\mathcal{S}(G)$  has the  $G$ -action induced by conjugation. It is easy to check that  $\rho$  is a  $G$ -map. For two elements  $t, t' \in \Pi(G, X)$ , we say  $t \leq t'$  if and only if  $X_t \subseteq X_{t'}$  and  $\rho(t) \supseteq \rho(t')$ . For  $t \in \Pi(G, H)$ , we define  $\hat{t}$  to be the minimal element in  $\Pi(G, H)$  such that  $X_t = X_{\hat{t}}$ . Then we define the set  $\hat{\Pi}(G, X)$  by

$$\hat{\Pi}(G, X) = \{\hat{t} \mid t \in \Pi(G, X)\}.$$

### 3. DEFINITION OF $\nabla W(\mathcal{D}_X)_{\text{proj}}$

Let  $X$  be a compact, connected, oriented smooth  $G$ -manifold of dimension  $n = 2k \geq 6$  satisfying the weak gap condition for  $\{e\}$  and

$$\mathcal{D}_X = (G, (-1)^k, w_X, \pi_X : \tilde{\Theta}_{X,+} \rightarrow \Theta_X, \rho_X : \Theta_X \rightarrow \mathcal{S}(G), Q_X)$$

the datum associated with  $X$  described in Section 1. We call a tuple  $\mathbf{M} = (M, B, q, \alpha)$  a  $\mathcal{D}_X$ -quadratic module if it satisfies the following.

- (1)  $M$  is a finitely generated projective  $\mathbb{Z}[G]$ -module.
- (2)  $B : M \times M \rightarrow \mathbb{Z}[G]$  is a nonsingular  $(-1)^k$ -Hermitian form. Thus,
  - (i)  $B$  is bilinear over  $\mathbb{Z}$ ,
  - (ii)  $B(x, y) = (-1)^k \overline{B(y, x)}$  for  $x, y \in M$ ,
  - (iii)  $B(x, ay) = aB(x, y)$  for  $a \in \mathbb{Z}[G]$ ,  $x, y \in M$ ,
- (3)  $q : M \rightarrow \mathbb{Z}[G]/(\Lambda(Q_X) + \mathbb{Z}[S_X])$  is a quadratic form associated with  $B$ . Thus,
  - (iv)  $q(ax) = aq(x)\bar{a}$  for  $a \in \mathbb{Z}[G]$ ,  $x \in M$ ,
  - (v)  $B(x, x) = \widetilde{q(x)} + (-1)^k \overline{\widetilde{q(x)}}$  in  $\mathbb{Z}[G]/\mathbb{Z}[S_X]$  for  $x \in M$ , where  $\widetilde{q(x)} \in \mathbb{Z}[G]$  is a lifting of  $q(x)$ .
  - (vi)  $q(x+y) - q(x) - q(y) = B(x, y)$  in  $\mathbb{Z}[G]/(\Lambda(Q_X) + \mathbb{Z}[S_X])$  for  $x, y \in M$ .
- (4)  $\alpha$  is a pair consisting of a  $(G \times \{\pm 1\})$ -map  $\tilde{\alpha}_+ : \tilde{\Theta}_{X,+} \rightarrow M$  and a  $G$ -map  $\alpha : \Theta_X \rightarrow M/2M$  such that the diagram

$$\begin{array}{ccc} \tilde{\Theta}_{X,+} & \xrightarrow{\tilde{\alpha}_+} & M \\ \pi_X \downarrow & & \downarrow \\ \Theta_X & \xrightarrow{\alpha} & M/2M \end{array}$$

commutes.

If  $\mathbf{M} = (M, B, q, \alpha)$  possesses a stably free  $\mathbb{Z}[G]$ -submodule  $L \subset M$  such that  $L = L^\perp$ ,  $q(L) = 0$ ,  $\text{Image}(\tilde{\alpha}_+) \subset L$  and  $\text{Image}(\alpha) \subset L/2L (\subset M/2M)$ , then  $L$  is called a *Lagrangian* of  $\mathbf{M}$  and  $\mathbf{M}$  is called a *null module*, where

$$L^\perp = \{x \in M \mid B(x, y) = 0 \ (y \in L)\}.$$

Let  $\mathfrak{Q}(\mathcal{D}_X)$  denote the category of  $\mathcal{D}_X$ -quadratic modules, where morphisms are isomorphisms. Then the notion of *direct sum* on  $\mathfrak{Q}(\mathcal{D}_X)$  is clear. Thus we have the Grothendieck group  $K_0(\mathfrak{Q}(\mathcal{D}_X))$  and the Witt group

$$W(\mathfrak{Q}(\mathcal{D}_X))_{\text{proj}} = K_0(\mathfrak{Q}(\mathcal{D}_X)) / \langle \text{null modules} \rangle.$$

Let  $\nabla\mathfrak{Q}(\mathcal{D}_X)$  be the full subcategory of  $\mathfrak{Q}(\mathcal{D}_X)$  consisting of all objects  $\mathbf{M} = (M, B, q, \alpha)$  such that

$$B(\Sigma_s - sx, x) = 0 \ \text{in } \mathbb{Z}_2[G] / \mathbb{Z}_2[G \setminus \{e\}] \ \text{for } s \in S_X, x \in M,$$

where

$$\Sigma_s = \sum \{\alpha(t) \mid t \in \Theta_X : \rho_X(t) \ni s\}.$$

We obtain the Grothendieck group  $K_0(\nabla\mathfrak{Q}(\mathcal{D}_X))$  and the Witt group

$$\nabla W(\mathcal{D}_X)_{\text{proj}} = K_0(\nabla\mathfrak{Q}(\mathcal{D}_X)) / \langle \text{null modules in } \nabla\mathfrak{Q}(\mathcal{D}_X) \rangle.$$

#### 4. COMPUTABILITY PROPERTY

Let  $\mathfrak{S}(G)$  be the subgroup category defined by J. A. Green [7]: namely its objects are subgroups of  $G$  and its morphisms are triples  $(H, g, K)$  such that  $H, K \in \mathcal{S}(G)$  and  $g \in G$  such that  $gHg^{-1} \subset K$ . Let  $\mathfrak{Ab}$  be the category of abelian groups: namely its objects are abelian groups and its morphisms are homomorphisms of groups. Let  $w : G \rightarrow \{\pm 1\}$  be a homomorphism and  $\mathcal{G}$  a family of subgroups of  $G$ . The notion of  $(w, \mathcal{G})$ -Mackey functor is similar to that of Mackey functor (cf. [13], [11]). A  $(w, \mathcal{G})$ -Mackey functor  $M = (M^*, M_*)$  is a bifunctor from  $\mathfrak{S}(G)$  to  $\mathfrak{Ab}$  such that  $M_*(H) = M^*(H)$  ( $= M(H)$ ) for  $H \in \mathcal{S}(G)$  and the following is satisfied.

- (1)  $c_{(H, g)*} = c_{(gHg^{-1}, g^{-1})}^*$  for  $H \in \mathcal{S}(G)$  and  $g \in G$ .
- (2)  $c_{(H, h)*} = w(h)id_{M(H)}$  for  $H \in \mathcal{S}(G)$  and  $h \in H$ .

(3)  $\text{res}_K^L \circ \text{ind}_H^L$  coincides with

$$\bigoplus_{KgH \in K \backslash L/H} \text{ind}_{H \cap gHg^{-1}}^K \circ (w(g)c_{(H \cap g^{-1}Kg, g)_*}) \circ \text{res}_{H \cap g^{-1}Kg}^H$$

for  $L \in \mathcal{G}$ ,  $H, K \in \mathcal{S}(L)$ ,

where  $c_{(H, g)_*} = M_*(H, g, gHg^{-1})$ ,  $c_{(H, g)^*}^* = M^*(H, g, gHg^{-1})$ ,  $\text{ind}_H^K = M_*(H, e, K)$ , and  $\text{res}_H^K = M^*(H, e, K)$ . In the case  $w$  is trivial, a  $(w, \mathcal{G})$ -Mackey functor is called a  $\mathcal{G}$ -Mackey functor. Moreover in the case  $\mathcal{G} = \mathcal{S}(G)$ , a  $\mathcal{G}$ -Mackey functor is called a Mackey functor.

**Theorem 4.1.** *Let  $X$  be as in Theorem 1.3. Then  $\nabla W(-)_{\text{proj}}$  canonically has the structure of a Mackey functor: namely there exists a Mackey functor  $M = (M_*, M^*) : \mathfrak{S}(G) \rightarrow \mathfrak{Ab}$  such that  $M_*(H) = M^*(H) = \nabla W(\mathcal{D}_H)_{\text{proj}}$ .*

Let  $\mathcal{F}$  be a family of subgroups of  $G$  closed under conjugation by all elements in  $G$  and under arbitrary intersections: namely  $gHg^{-1} \in \mathcal{F}$  for all  $H \in \mathcal{F}$  and  $g \in G$ , and  $H \cap K \in \mathcal{F}$  holds for all  $H, K \in \mathcal{F}$ . A  $(w, \mathcal{G})$ -Mackey functor  $M$  is called  $\mathcal{F}$ -computable if the induction homomorphism

$$\text{Ind} : \lim_{\rightarrow \mathcal{F}} M(-) \rightarrow M(G)$$

and the restriction homomorphism

$$\text{Res} : M(G) \rightarrow \lim_{\leftarrow \mathcal{F}} M(-)$$

are both isomorphisms. If  $\mathcal{U}$  is a set of prime integers then we denote by  $\mathcal{U}'$  the multiplicatively closed subset of integers generated by 1 and all prime integers  $q \notin \mathcal{U}$  dividing  $|G|$ . A  $(w, \mathcal{G})$ -Mackey functor  $M$  is called  $\mathcal{F}$ -hypercomputable if  $\mathcal{U}'^{-1}M$  is  $\mathcal{F}^{\mathcal{U}}$ -computable for all  $\mathcal{U}$  as above, where

$$\mathcal{F}^{\mathcal{U}} = \{K \in \mathcal{S}(G) \mid K \supseteq H, H \in \mathcal{F}, K/H \text{ has } p\text{-power order for some } p \in \mathcal{U}\}.$$

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